AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 3

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

$v(3) = 0$

$v(4.5) = -1$ v is neg

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

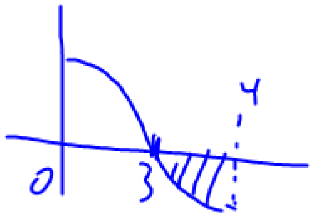
Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$? $\rightarrow 2.386$

~~(d) What is the position of the object at time $t = 4$?~~

$$s(x) = \int_0^4 v(t)\,dt = \cancel{-0.477}\ \ 1.43$$



$$\int_0^3 v(t)\,dt \quad 1.909$$

$$2 + \int_0^4 v(t)\,dt = 3.43$$

0.477

$$\int_3^4 v(t)\,dt \quad -0.477$$

(a) $a(4) = v'(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right)$

$= -\frac{\pi}{6}$ or -0.523 or -0.524

1 : answer

(b) On $3 < t < 4.5$:

$a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right) < 0$

Statement I is correct since $a(t) < 0$.

Statement II is correct since $v(t) < 0$ and $a(t) < 0$.

$$3 \begin{cases} 1: \text{ I correct, with reason} \\ 1: \text{ II correct} \\ 1: \text{ reason for II} \end{cases}$$

(c) Distance $= \int_0^4 |v(t)|\,dt = 2.387$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$

$x(0) = 2$

$x(4) = 2 + \frac{9}{2\pi} = 3.43239$

$v(t) = 0$ when $t = 3$

$x(3) = \frac{6}{\pi} + 2 = 3.90986$

$|x(3) - x(0)| + |x(4) - x(3)| = \frac{15}{2\pi} = 2.387$

3:

- 1 : limits of 0 and 4 on an integral of $v(t)$ or $|v(t)|$

  or

  uses $x(0)$ and $x(4)$ to compute distance
- 1 : handles change of direction at student's turning point
- 1 : answer

  0/1 if incorrect turning point or no turning point

(d) $x(4) = x(0) + \int_0^4 v(t)\,dt = 3.432$

$$2 \begin{cases} 1: \text{ integral} \\ 1: \text{ answer} \end{cases}$$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$

$x(4) = 2 + \frac{9}{2\pi} = 3.432$

OR

$$2 \begin{cases} 1: \ x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + C \\ 1: \text{ answer} \end{cases}$$

0/1 if no constant of integration

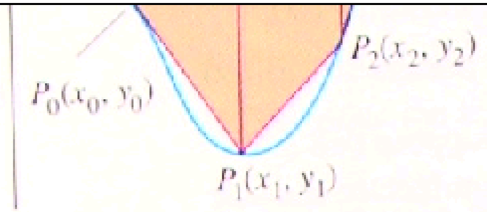
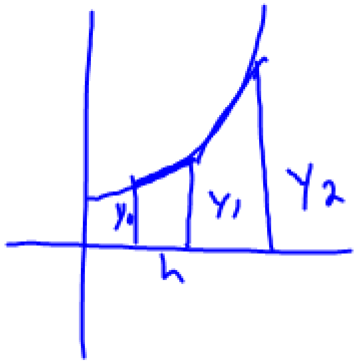


Figure 5.31 The trapezoidal rule approximates short stretches of the curve $y = f(x)$ with line segments. To approximate the integral of f from a to b, we add the "signed" areas of the trapezoids made by joining the ends of the segments to the x-axis.

The region between the curve and the x-axis is then approximated by the trapezoids, the area of each trapezoid being the length of its horizontal "altitude" times the average of its two vertical "bases." That is,



$$\int_a^b f(x)\,dx \approx h \cdot \frac{y_0 + y_1}{2} + h \cdot \frac{y_1 + y_2}{2} + \cdots + h \cdot \frac{y_{n-1} + y_n}{2}$$

$\frac{1}{2}h(b_1 + b_2)$

$$= h\left(\frac{y_0}{2} + y_1 + y_2 + \cdots + y_{n-1} + \frac{y_n}{2}\right)$$

$$= \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right),$$

where

$$y_0 = f(a), \quad y_1 = f(x_1), \quad \cdots, \quad y_{n-1} = f(x_{n-1}), \quad y_n = f(b).$$

This is algebraically equivalent to finding the numerical average of LRAM and RRAM; indeed, that is how some texts define the Trapezoidal Rule.

Figure 5.3[...] mation of the [...] $y = x^2$ from [...] overestimate[...]

Table 5.4

The Trapezoidal Rule

To approximate $\int_a^b f(x)\,dx$, use

$$\boxed{T = \frac{h}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right),}$$

where $[a, b]$ is partitioned into n subintervals of equal length $\boxed{h = (b - a)/n.}$

Equivalently,

$$T = \frac{\text{LRAM}_n + \text{RRAM}_n}{2},$$

where LRAM_n and RRAM_n are the Riemann sums using the left and right endpoints, respectively, for f for the partition.

EXAMPLE 1 Applying the Trapezoidal Rule

Use the Trapezoidal Rule with $n = 4$ to estimate $\int_1^2 x^2\,dx$. Compare the estimate with the value of NINT $(x^2, x, 1, 2)$ and with the exact value.

SOLUTION

Partition $[1, 2]$ into four subintervals of equal length (Figure 5.32). Then evaluate $y = x^2$ at each partition point (Table 5.4).

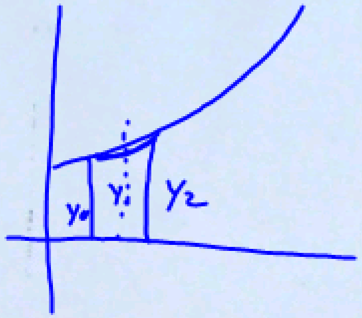
Using these y values, $n = 4$, and $h = (2 - 1)/4 = 1/4$ in the Trapezoidal Rule, we

Do #1-6

on 5 verify w/ $\underbrace{\text{Part 2 of Fundamental Thm}}_{\text{take integral}}$

$$\boxed{\tfrac{1}{3}h(2ah^2 + 6c)}$$

3. Write y_0, y_1, and y_2 in terms of a, b, c, and h. Then write $y_0 + y_2$ in terms of a, b, c, and h.



$$y_2 = ax^2 + bx + c \rightarrow ah^2 + bh + c$$
$$y_1 = c \longrightarrow c$$
$$y_0 = ax^2 - bx + c \rightarrow ah^2 - bh + c$$

4. By appropriate algebra, show that the area of the region under the parabola may be found from the y-values and h alone, namely,

Area $= \frac{1}{3}h(y_0 + 4y_1 + y_2)$.

$$\tfrac{1}{3}h((ah^2 + bh + c) + 4c + (ah^2 - bh + c))$$
$$\tfrac{1}{3}h(ah^2 + bh + c + 4c + ah^2 - bh + c)$$
$$\tfrac{1}{3}h(2ah^2 + 6c)$$

Simpson's rule. Suppose that $\int_a y\,dx$ be evaluated using Simpson's rule with n increments, each of width h. Write a form the integral in terms of h and the values of

7. Program your grapher to evaluate integrals Simpson's rule. Evaluate the integral in Pr using $n = 100$ increments. How does the an compare with the exact answer, 295,391.80

8. Evaluate the integral in Problem 5 using the trapezoidal rule and using midpoint Riemann sums, each with $n = 100$. Which of the three approximations is closest to the actual value, and which is farthest away?

9. Why must n be an *even* number for Simpson's to be used?

at the vertex at the origin.)

$f(0)$, and $y_2 = f(h)$. By doing the
at the area of the region under the
o $x = h$ is
).

$$\frac{2ah^3}{3} + \frac{6ch}{3}$$

x

$$h - \left(-\frac{ah^3}{3} + \frac{bh^2}{2} - ch\right)$$

$$\boxed{2ch} = \frac{1}{3}h(2ah^2 + 6c)$$

~~n terms of a, b, c, and h.~~
terms of a, b, c, and h.

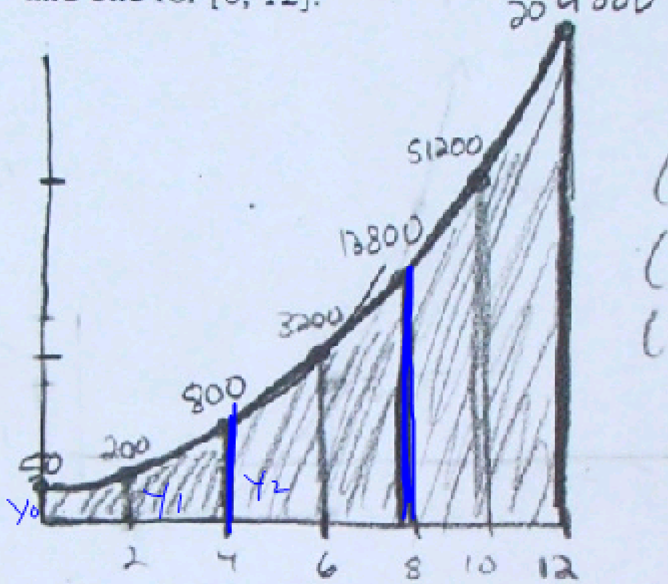
$h + c$

$h + c$

Partition the interval $[0, 12]$ into six subintervals of width $h = 2$. Then use the result of Problem 4 to estimate

$$\int_0^{12} 50 \cdot 2^x \, dx$$

by using three parabolas, one for $[0, 4]$, one for $[4, 8]$, and one for $[8, 12]$.



$(0,4) = 2200$

$(4,8) = 35200$

$(8,12) = 563200$

$$\frac{1}{3}h(y_0 + 4y_1 + y_2)$$

$$\frac{1}{3} \cdot 2\left(50 \cdot 2^0 + 4 \cdot 50 \cdot 2^2 \ldots\right.$$

6. The technique you used in Problem 5 is called **Simpson's rule.** Suppose that $\int_a^b y\,dx$ is to be evaluated using Simpson's rule with n equal increments, each of width h. Write a formula for the integral in terms of h and the values of y_i.

$$\frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right)$$

7. Program your grapher to evaluate integrals by Simpson's rule. Evaluate the integral in Problem 5 using $n = 100$ increments. How does ...

5.5 # 1-6 (3), 7-12, 13-18 (3), 29, 30, 31-36
MC